

## Key Point

- A deep learning (DL) model exhibits superior prediction skill in the tropical Pacific compared to a linear inverse model.
- Implement a novel inflation technique by incorporating noise from hindcast experiments to address DL signal damping.
- Data assimilation (DA) on a sparse network of observations accurately reconstructs the monthly upper ocean spatial fields.
- The superior reconstruction skill of the deep learning model stems from its enhanced prediction skill.

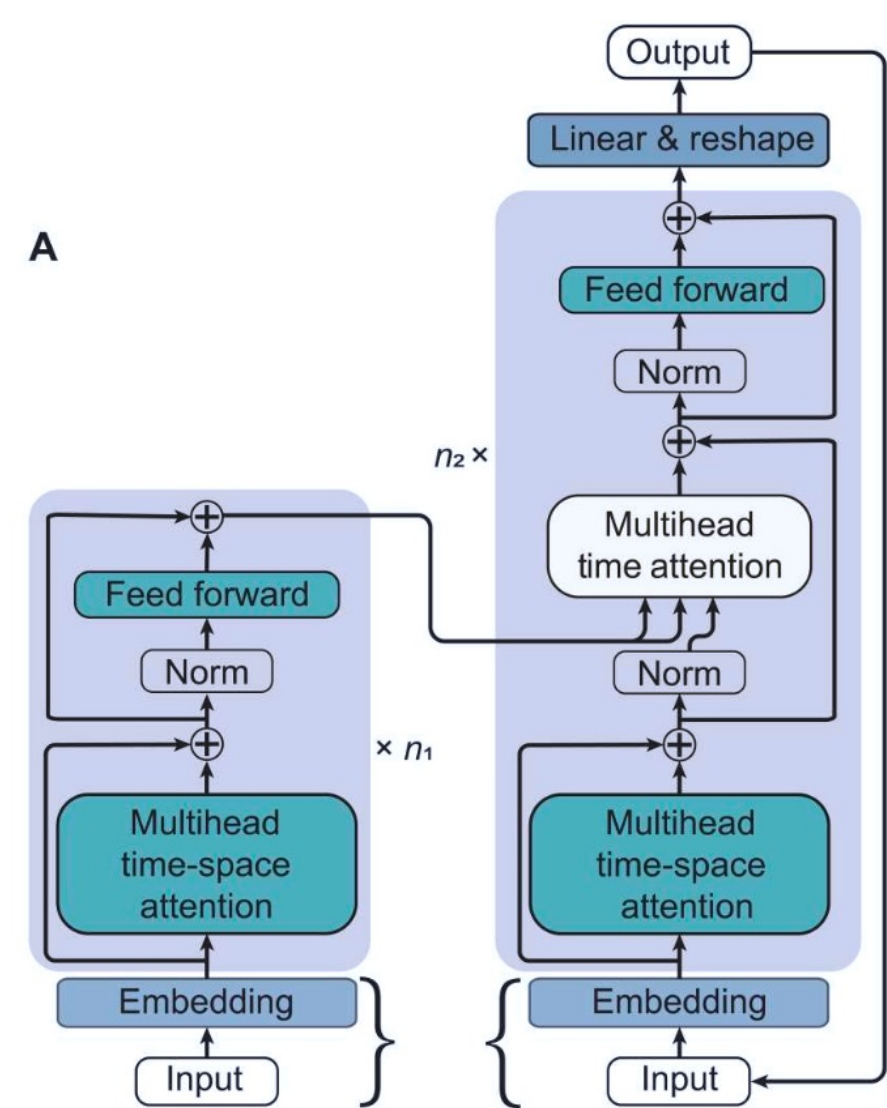
## Baseline: Linear Inverse Model

$$\frac{dx}{dt} = \mathbf{L}x + \xi \quad \mathbf{L}: \text{deterministic Matrix}; \xi: \text{random noise};$$

$$\mathbf{G}_\tau = \mathbf{C}(\tau)\mathbf{C}(0)^{-1} \quad \mathbf{C}(\tau) = \langle \mathbf{x}^T(\tau)\mathbf{x}(0) \rangle \quad \mathbf{G}_\tau = \exp(\mathbf{L}\tau)$$

x: state vector (PCs from EOF).

## Deep Learning Model (Zhou&Zhang, 2023)



$$\mathbf{X}_{t+1:t+12}^{\text{out}} = \mathbf{DL}(\mathbf{X}_{t-12:t}^{\text{in}})$$

$\mathbf{X}$ : Surface wind stress, upper ocean temperature.

**Model structure:** self-attention and encoder-decoder structure.

**Training:** trained on CMIP6, verified on Simple Ocean Data Assimilation (SODA) products, tested on Global Ocean Data Assimilation System (GODAS) reanalysis (Independent verification).

**Loss:** all fields RMSE.

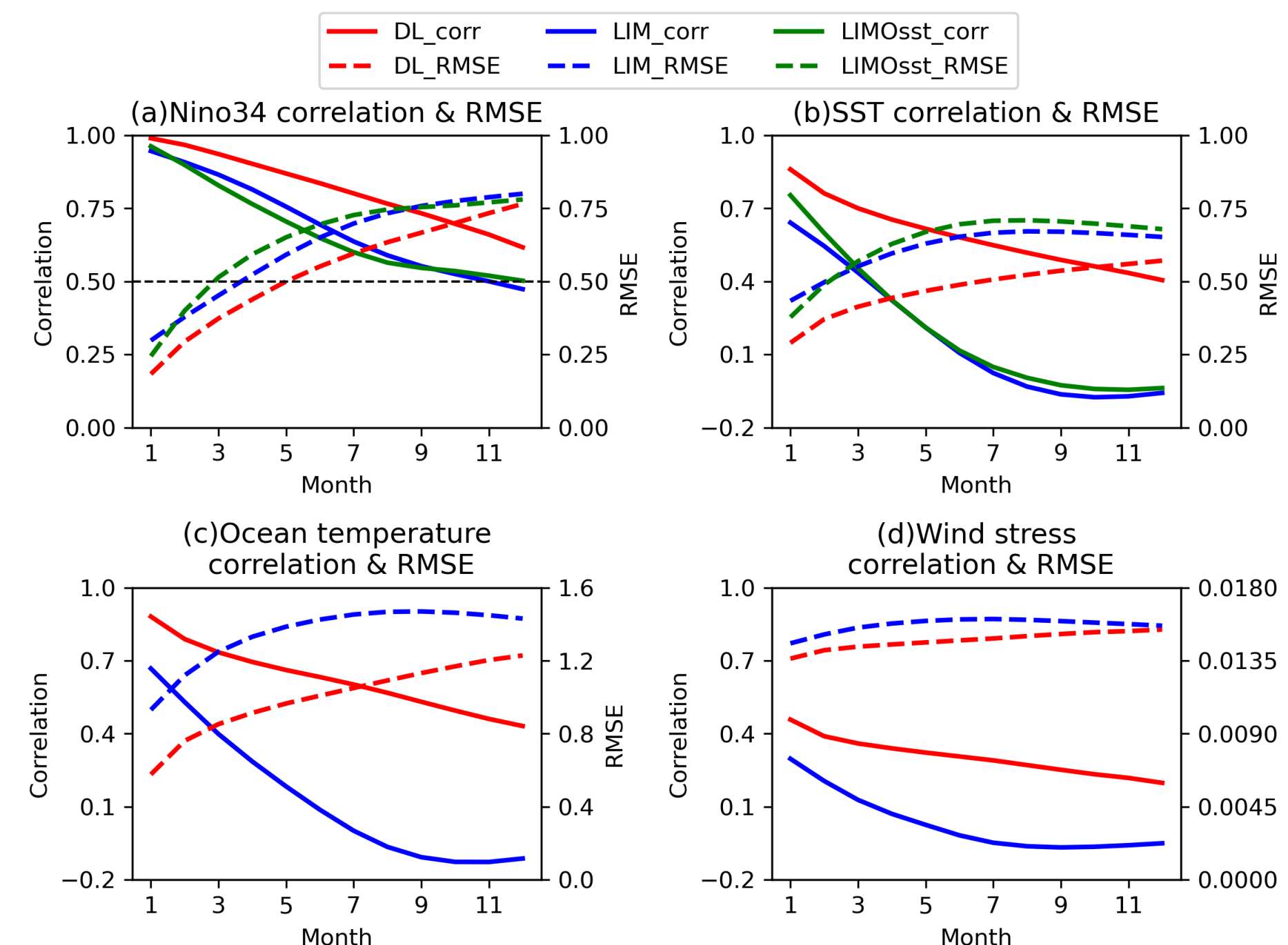
**Advantages:** Capture the nonlinear dynamics; No need to truncate data; Trained on larger ensemble data.

## DA Method: Ensemble Kalman Filter

$$\mathbf{x}_a = \mathbf{x}_p + \mathbf{K}[\mathbf{y} - \mathcal{H}(\mathbf{x}_p)] \quad \mathbf{x}_{p,t+1} = \mathcal{M}(\mathbf{x}_a, t)$$

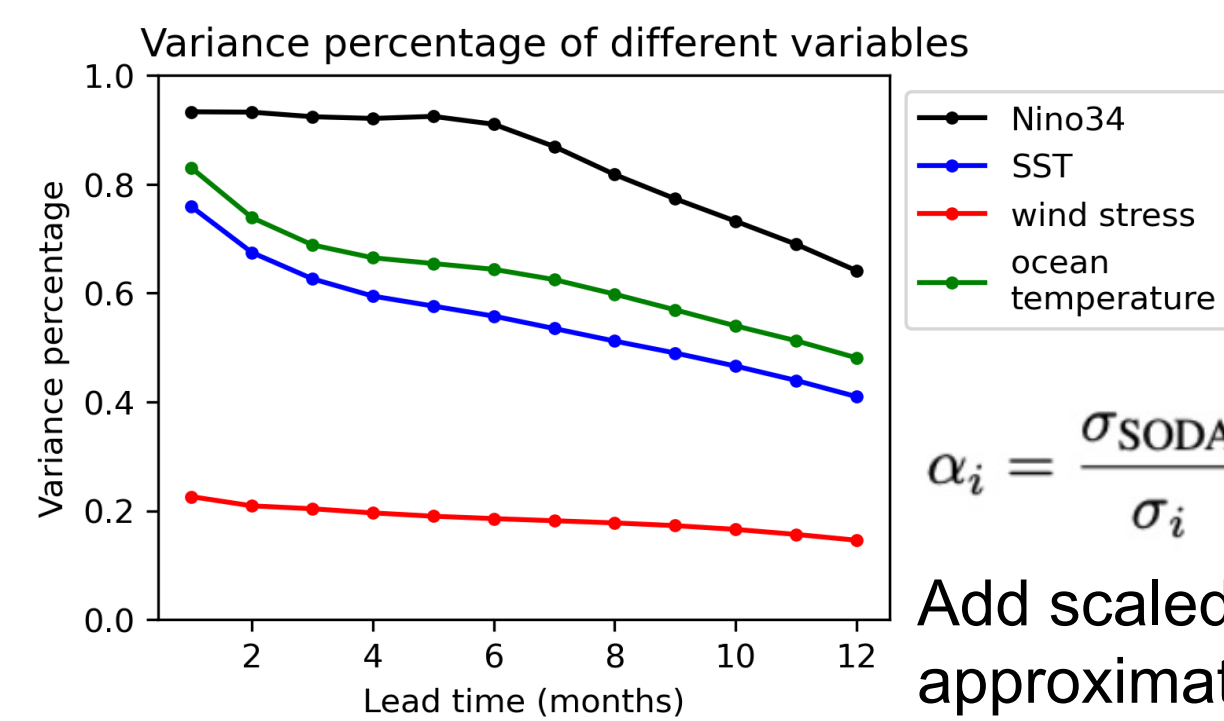
$\mathbf{x}_a$ : posterior;  $\mathbf{x}_p$ : prior;  $\mathbf{K}$ : Kalman gain;  $\mathbf{y}$ : Observation;  $\mathcal{H}$ : observation operator.  $\mathcal{M}$ : Model operator.

## Prediction Skill



DL forecasts consistently outperform both the LIM and LIMOst (SST only LIM) forecasts across all variables and at all lead times.

## DL Model Ensemble Inflation



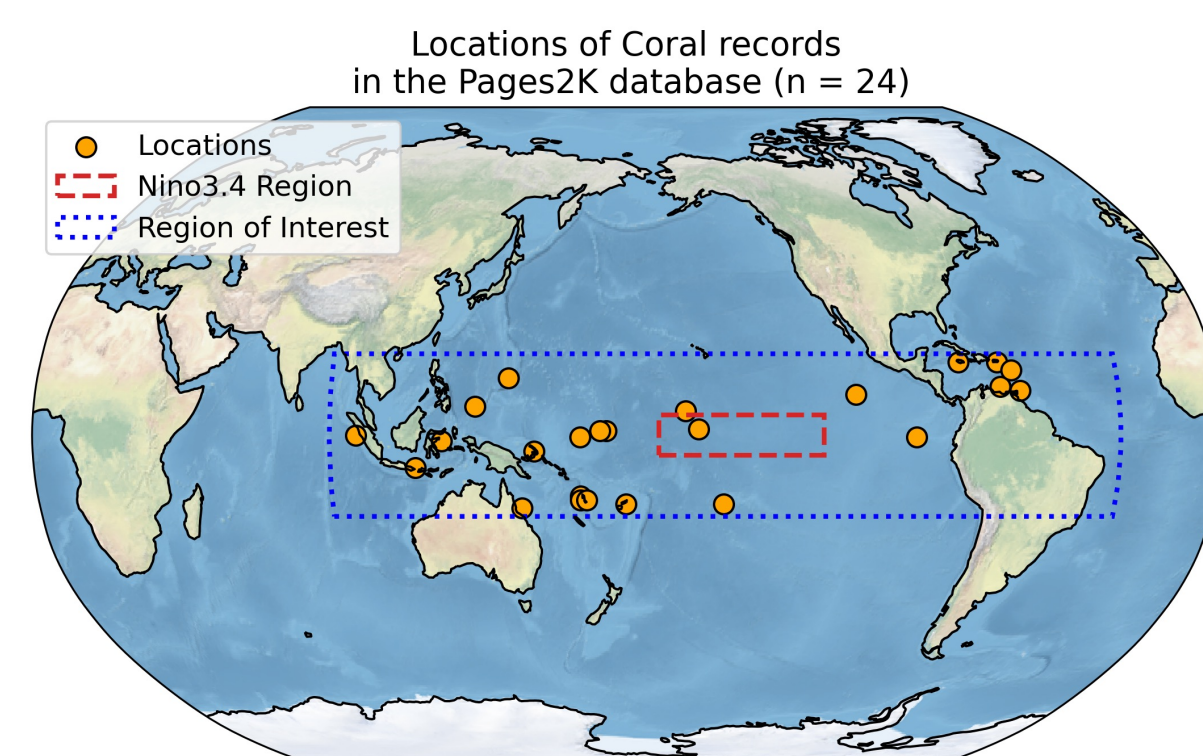
DL models tend to lose error associated with the unpredictable signal.

$$\eta_{m,l,i} = \alpha_i(\mathbf{x}_{m,l,i}^{\text{true}} - \mathbf{x}_{m,l,i}^{\text{out}})$$

$$\alpha_i = \frac{\sigma_{\text{SODA}}}{\sigma_i} \quad \mathbf{x}_{t+1} = \mathbf{x}_{m,l}^{\text{out}} + \eta_{m,l}$$

Add scaled noise from hindcasting to better approximate errors.

## Pseudo Proxy Experiment

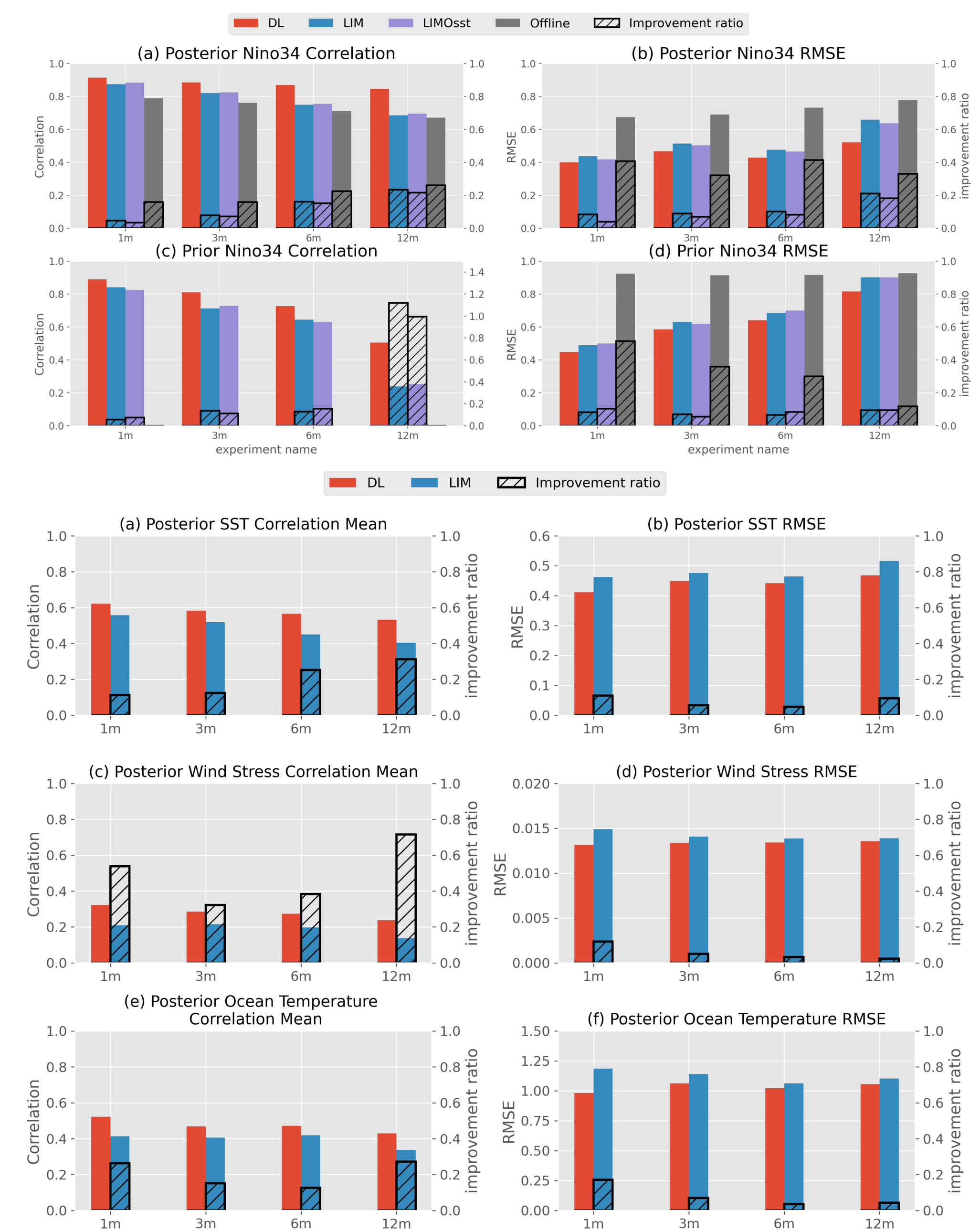


$$y_{\text{avg},N} = \frac{1}{N} \sum_{i=k+1}^{k+N} y_i$$

$$y'_{\text{avg},N} = y_{\text{avg},N} + \zeta$$

Average SST at coral locations over periods of 1, 3, 6 and 12 months and add white noise to mimic coral situation.

## Data Assimilation Result



- DL model outperforms the others by around from 10 % to 30 % in reconstructing the Nino3.4 index.
- Improvement using the DL model increases with observation averaging time.
- In terms of skill across the entire domain, the DL model outperforms the LIMs for all observation averaging times and variables, most notably for correlation, and less so for RMSE.
- These improvements reflect a combination of the forecast-skill improvements, which better retain the memory of past observations, and improved spatial covariance, which spread information from the sparse network of observations.